SPE 115541

The Consideration and Applicability of Game Theory to Reserve Valuation

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This paper was prepared for presentation at the 2008 SPE Asia Pacific Oil & Gas Conference and Exhibition held in Perth, Australia, 20–22 October 2008.

Abstract

Petroleum engineers follow guidelines set forth by the SPE/WPC/AAPG/SPEE for reserve classification when estimating reserves. Once these classifications have been defined, the reserves are risked accordingly. This classification allows uncertainty and risk to be incorporated into the reserve estimate itself, but not necessarily the dynamics of the chosen strategies for recovering and marketing the reserve.

Because of the many unknowns and uncertainties inherent in accurately identifying a reserve amount, the process includes an element of chance and, therefore, incorporates probability theory. By the same logic, the process of identifying a reserve value, which is realized by implementing a recovery strategy, should incorporate game theory.

Probability theory deals with chance, while game theory deals with strategy. Probability theory assists with the classification of the risk associated with the actuality of the reserve amount, while game theory assists with the classification of the strategy associated with the recovery of the reserve, which defines the fair market value of the reserve.

Estimating a fair market value for a reserve must incorporate a whole new set of parameters. A consideration that should be incorporated into a reserve valuation is the entities attempting to recover and market the oil and gas reserves through their designed and implemented strategies. These entities' decisions will affect the efficiency of recovering the reserves. These decisions are typically made by more than one decision maker and they may be in conflict and they may be negotiated or non-negotiated.

For example, consider operators that are small independents with capital constraints. Decisions and strategies may be decided upon that are more for short term financial reasons, rather than efficient production of the reserves. Additional examples might include the involvement of unstable governmental regimes that are unable to protect the recovery operations or companies with dynamic economic focus that lower priorities for recovering specific reserves. All of these different "player strategies" can affect the time and cost of recovering the estimated reserves and, ultimately, affect the fair market value of the defined reserves.

The payoff is defined as a proportionate amount of the total value of the reserves to each player. The payoff is, therefore, the variably defined fair market value. There are three players; the owner of the reserves (the lessor), the operator of the recovery operations (the lessee) and the service providers. All three players are concerned with having their maximum payoff from the recovery of the defined reserves. The lessor of the reserves, by virtue of its lease to the lessee, has assigned its associated costs to the lessee for a defined proportionate share of the defined reserve. The lessee assumes all costs. Its payoff is the total value of the reserves, less the costs and royalties. The lessee's costs are the service providers' payoff.

This paper discusses practical concepts of game theory for the inclusion of "player affects or strategies" that may include "conflicting decision makers" and "negotiated" or "non-negotiated strategies" in the methodology for arriving at a fair market value of estimated reserves.

Introduction

Probability theory deals with chance, while game theory deals with strategy (Figure 1). Probability theory can assist with the classification of the risk associated with the actuality of a reserve estimate. By a similar concept, game theory can assist with the classification of the strategy associated with the design and implementation of the recovery of those reserves. The implementation of a chosen strategy should ultimately be considered in defining the value of the reserve or the value or fair market value (FMV). Thus, by considering game theory concepts in the valuation of reserves, more accuracy may be realized.

Estimating a net present or fair market value for reserves must incorporate a whole new set of parameters. For example, the cost and length of time to recover these reserves must be considered. A consideration that should be incorporated into a reserve valuation is the companies or individuals attempting to recover and market the oil and gas reserves. These entities' decisions will affect the efficiency of recovering the reserves. These decisions may be made by more than one decision maker and may be in conflict. For example, consider operators that are small independents with capital constraints. Decisions and strategies may be decided upon that are more for short term financial reasons, rather than efficient production of the reserves. Additional examples might include unstable governmental regimes that halt or hinder production or destroy facilities or small interest owners held hostage by financially stressed larger interest owners. Companies with dynamic economic focus that lower priorities for recovering specific reserves could also affect the economic scenario for recovering these reserves. All of these different "players" can affect the time and cost of recovering the estimated reserves and, ultimately, affect the recovery efficiency, fair market value and net present value estimate of the reserves.

Value of Defined Reserves or Fair Market Value

A fair market value analysis initially establishes a range of values for a given reserve. Ultimately, and in general, a fair market value will be defined as the price at which a willing seller and a willing buyer agree.

"Fair market value is the price for property which would be agreed upon between a willing and informed buyer and a willing and informed seller under usual and ordinary circumstance. It shall be the highest price estimated in terms of money which property will bring if exposed for sale on the open market with reasonable time allowed to find a purchaser who is buying with knowledge of all the uses and purposes to which the property is best adapted and for which it can be legally used." (Louisiana Revised Statute 47:2321)

This paper is concerned with the analysis of the range of values for a fair market value or the consideration of various aspects that might affect the final agreed upon price of a reserve before it actually happens.

One of the main problems that are inherent in the establishment of a value of a reserve is the necessary idealized approach required. If all aspects that could affect the ultimate value of the reserve are not considered then the accuracy of the value established will be diminished, even if it is the agreed upon price established by a willing buyer and a willing seller. If an unforeseen circumstance affects the recovery of a defined reserve then the value of the reserve that was set before the circumstance occurred will be inaccurate. That is the nature of the game; luck; a coin toss. "I'm glad I bought that reserve when I did." Or, "I sure am glad we sold those reserves when we did." Then on the other side, "I can't believe we paid that much for those reserves."

Of course, one can never foresee the future, but one may try. There are many theoretical tools available that can assist with the consideration of uncertainties and unknowns in economic scenarios. Statistics, for example, assigns probabilities to certain situations occurring in the future using established probability functions. Game theory can assist with the analyses of strategies available to recover the reserves and bring them to market, where the final value will be realized. Through this analysis, the consideration of the players and their circumstances are accounted for. This is only one aspect involved in the establishment of a value. It is one aspect that can dramatically affect the ultimate value, however.

The method for the establishment of a value or a fair market value for a reserve can take many forms. In its simplest forms, it can be based on a value "in-the-ground", a return of investment over a specific period of time, a net present value (NPV) at a specific discount rate, a discounted cash flow (DCF) value or an average of all of these. The nature of these methods is sufficient to render the estimated value inherently imprecise. Gustavson and Murphy defined various methods of establishing FMV and they addressed adjustments for certain risks and uncertainties. They outline and define "Dollars per Barrel of Oil in the

Ground," "Cumulative Cash Flow," "Targeted Internal Rate of Return" and "Risked Present Worth." (Gustavson and Murphy, 1989)

Only the "Risked Present Worth" method actually contains a defined risk element. In this method, they determine a risk via establishing ranks derived from various questions concerning the reserve risks, the operational risks and the financial risks. The reserve risks are more apt to be handled using probability theory, while the operational and financial risk factors are more apt to be handled somewhat by game application because of the necessary strategic planning involved. It should be noted that when analyzing the strategies to recover the reserves, one must assume that the reserves are defined.

The Gustavson and Murphy methods enlighten one as to the inherent difficulties with attempting to establish a value to an accepted reserve volume. Excluding the reserve risks, when Gustavson and Murphy calculate the "Risked Present Worth" reserve value, they consider an operator's experience, an operator's reputation and ability, an operator's effectiveness, the quality of the mechanical equipment, the complexity of the operations and equipment, an operator's working interest in the property, an operator's ability to meet future financial needs, an operator's ability to improve production, and whether or not a field visit was made, in their risk analysis of the operations. They also consider contract conditions, exposure to plugging liabilities, location of gas reserves, whether working interest was purchased, and if there is a significant royalty position concerned. These specific considerations are important, but they do not encompass the entire scope of unforeseen possibilities. This would be impossible. There are few tools that attempt to analyze all possibilities and a buyer and a seller will probably have very different assessments of the importance toward their individual affects on a value. The application of gaming is one of the tools available for these considerations.

Analysis of Recovery Strategies

Rapoport described game theory is that branch of mathematics that is concerned with rational decision. He goes on to say that it attempts to abstract life situations, but at best, only essential parameters can realistically be considered, rather than an entire situation, and those parameters considered must be initially identified. (Rapoport, 1966) By identifying primary parameters involved in a situation, a model of possible strategies may be envisioned and a game decision process made. Possible situations may include cooperative and non-cooperative players, dominating conditions or conflicting interpretations of value.

Regardless of the multitude of possible situations, the game may initially be defined simplistically as to conditions required to recover the pre-determined reserves. When initially simplistic conditions are made, more detailed refinements will evolve from the unique situation that the game is being applied to. The situations are infinite as are the strategies available, yet the concept of gaming may be applied in all cases.

Defining the Game

When initially considering the applicability of a game theory type analysis toward refining the valuation of reserves the sheer magnitude of the number of different situations is apparent. There could be single player games or "games against nature", two-player games, or multiple player games. A single player game can evaluate a reserve value that has many uncertainties toward the recovery strategy. The parameters can be defined and a single player game analysis performed. The parameters might include a water-flood or a gas-flood. Mars and Rinat outlined a game theory application for hydraulic fracture design (Mars and Rinat, 2004).

Two-player games might be applied toward two conflicting entities involved in the recovery of the reserves. The individual strategies that must coincide with each other can be evaluated for their affects on the value of the reserve. The same applicability is available for multiple player games. The importance is how the game is defined. Games, as one typically thinks of them, such as card games and board games, are all played by sets of rules. They also are played chronologically, with one player making a move and the next player moving after that with knowledge of the previous player's move. This type of game is also known as the extensive form of a game. Game analysis reduces extensive games to ones in a normal form, where moves or strategies are decided upon without prior knowledge of the other player(s) moves or strategies.

Therein is the value of game analysis. If one is able to properly define a game and able to identify eventual or expected outcomes resulting from various strategies, one would be able to more accurately define a value.

Reserve Value Game Definition

Players might be are initially defined as anyone or anything directly attempting to recover the remaining oil and gas (the reserve) which has been previously defined, that may realize a profit from the recovery of the reserve. Other parameters or players to be considered in sub-plots or sub-games could include anyone or anything indirectly involved with the recovery of the reserve that may realize a payoff from the recovery. When considering a particular defined game, only players may realize a payoff (from the value or fair market value of the eventual ultimate recovery of the reserve). However, those players do not have to be the primary ones. For example, a small working interest participant could apply gaming concepts just as easily to its proportionate share under its proportionate conditions as any primary player. The game just has to be defined for the specific application.

In general, the primary players involved in a primary game would be as follows:

1) Lessor
2) Lessee
3) Service companies

For a primary game, different strategies for the recovery of a reserve would normally be considered primary for the different players and these different strategies will affect the overall value of the reserve. With no other considerations being made, the lessee would have a recovery strategy based upon the fact that he has no costs (a royalty position); the lessor would have a recovery strategy based on the fact that he has costs (a working interest) and the service provider would have a recovery profit based on the costs to recover the reserve. However, all three players interact to bring the reserve to market, where the final value is realized. The lessee realizes the gross price paid, the lessor realizes the net price paid and the service provider realizes the end of its value. The value considered herein is the established value to the sellers.

The point is that the value of the reserves is affected by the players involved in the recovery of the reserves. The amount of the reserves has not changed, yet the value of those reserves is dynamic depending on the players involved with the recovery and delivery to market and their particular involvement in the process.

Example Situations

A Simple Game Analogy

A simple example of a game strategy might be a basketball tournament based on the winner being the best out of seven games. The seven games represent the reserve. The end of the game is represented by winning four games out of seven (7), which is 4/7 or 57.1% and this represents the percent recovery of the seven games. The number of games ultimately played to reach the 57.1% recovery efficiency affects the value of the recovery. Only four games could be played if the ultimate winner won every one of the first four games played or a total of seven games could be played if the final game was the determination of the winner after each team had won three games each.

The two (2) teams playing the seven potential (7) game series are the primary players. The sub-players are represented by the owners of the two teams, the players of the two teams, the advertisers of the games, the viewers (fans), and the television schedulers. Each player and sub-player has an ultimate best pay-off from the outcome of the playing of the four to seven games played. Each could influence the game played even if in a negligible way. Therefore, all could be considered in the analysis of the decision tree.

The importance of all of this is that even though a reserve figure has been established including associated risk of the defined reserves, the ultimate FMV must include the entities recovering the reserves, bringing them to market and collecting the payoff. Just because there are recoverable reserves in the ground that have been risked accordingly for the type of reserve they are, does not mean that a FMV may be established or booked in actuality without taking into consideration the parameters required to bring those reserves to market for fair trade and value.

Let’s go back to the basketball analogy. What is the highest payoff of the tournament if all players and sub-players defined above are included? A matrix of this scenario results in a seven game series with one of the teams winning with a record of 4-0, 4-1, 4-2 or 4-3. A full seven game series (4-3) provides the most payout to all entities involved. The losing team receives a payout of three won games. The winning team receives the best outcome of winning four games. The advertisers realize advertising over seven games versus four games. The owners of both teams realize revenue from playing seven games versus four games. The viewers are able to watch seven games rather than four games. The fans of the winning team get to watch their team win four games, while enjoying the benefit of entertainment over seven games. The fans of the losing team get to watch their team win a maximum of three games. The television schedulers get to book advertising revenue of seven games versus four games and do not have to expend extra energy to reschedule other viewing after the series goes to less than seven games.

Small Interest Holder versus Interest Holder

One example of a two-player game affecting the fair market value payout value might be a small interest holder of reserves (SI) in conflict with a large interest holder of those same reserves (LI). SI owns ten percent (10%) of the reserves and LI owns ninety percent (90%) of the reserves. At the present (or beginning of the game application), all wells capable of recovering the reserves are shut-in needing repairs. Also, LI does not have sufficient resources to pay for its proportionate share of the costs of the repairs. SI does have sufficient resources to pay for its proportionate share. The only reason that wells are shut-in and not producing is because LI cannot pay for its proportionate share of the costs.

Both SI and LI operate the reserves under an operating agreement, which states among other things, that if an authorization for expenditure (AFE) is presented, then both parties must elect to either consent or non-consent to the proposal. If they consent, then they must pay their proportionate share of the AFE estimate. If one of the parties decides to go non-consent, while the other consents, then the consenting party has the right to pay for the non-consenting party's proportionate share. In return, the consenting party paying for the non-consenting party receives a 500% penalty of those costs from the non-consenting party's interest. The non-consenting party also loses its right to participate in any future work until the paying consenting party realizes its payback of 500% of the costs from the non-consenting interest. However, it still technically owns the interest and would regain full rights once the penalty is paid back. Of course, the paying consenting party has now undertaken more risk. Therefore, if SI is the consenting party, it faces the predicament of potentially going into the same situation as the non-consenting party, LI, by having some unforeseen expensive problem arising from the repair work. Being a small interest owner could possibly mean that its resources are limited as well.

The value of the reserves is less than it could be in their present state. By definition, proved producing reserves have more value than proved non-producing reserves. In order to bring the FMV of the reserves up to its full potential, additional risk in the form of repair work to the wells, must be undertaken. Therefore, the players of the game are affecting the value of the payout or the fair market value of the reserves; not only the total payout, but their own share of the payout. A willing buyer would expect a discount for buying non-producing reserves. In order for the buyer to realize returns from its investment, it must have the reserves producing. In addition, the potential buyer probably sees red flags concerning other parameters, if it is aware of the financial distress of LI (or even if it is not).

There are several strategies available to resolve this conflict and there are several game applications that could be made. Each strategy affects the value of the payout to each player. SI or LI could sell its interest for the present FMV, which is reduced from its full potential because the reserves are not producing. SI could consent that the wells be repaired and pay for non-consenting LI's share of the costs. SI takes on additional risk for the additional added value by virtue of LI's non-consent penalty and LI potentially does not lose its interest. LI could sell a portion of its interest to pay for its share of the repair work.

As an example for the application of game theory to assist with reserve valuation, a specific game will be defined concerning this situation and will be limited for clarification. In definition of the game, the element of risk and the bargaining attributes of the players are removed and the payouts of consenting or non-consenting are reviewed only. In this case, it would be assumed that the repair work would be successful and that a greater FMV would be realized by redefining the reserves from proved non-producing to proved producing. Therefore, the game will be defined as a simple analysis of consent/non-consent strategies.

A simple matrix is laid out as shown in Table 1, where

$a < 0.1$
$b < 0.9$
$c > 0.1$
$d < b$
$e < a$
$f > 0.9$

Both parties have the option of consenting or non-consenting. If one consents and the other non-consents, the consenting party also has the option of taking the 500% penalty enticement for the other party's interest obligation. If both non-consent, the reserves' FMV remains at the reduced value for proved non-producing classification. This would be represented by SI receiving "a" (which is less than 10%) and LI receiving "b" (which is less than 90%). If SI consents while LI non-consents, SI receives "c" (which is greater than 10%) while LI receives "d" (which is less than b). If SI non-consents while LI consents, SI receives "e" (which is less than 10%) while LI receives "f" (which is greater than 90%). If both parties consent, they both receive full value or rather SI receives 10% and LI receives 90%. The defined matrix in a payout graph results in Figure 1.

If both parties decide to go non-consent, then the full potential value of the reserves is not realized for either. If one goes consent while the other goes non-consent, then the work is performed and the total value of the reserves is realized. However, the value to the consenting player is increased by reducing the non-consenting player's value. If both parties consent, then the full potential value is realized for both players. Notice that the only strategy where the full potential value is realized by both is where both players consent to the work.

As presented and defined with no element of risk or any other intrinsic parameters considered, this game may be described as a non-zero two-person game. No negotiation is required by either party and decisions can be made prior to knowing what the other party is going to do. Therefore, one can identify the Nash Equilibrium point, described by John Nash, as the consent-consent strategy with the maximum payoff to both players. For any other strategy, either both players realize a lower payoff or one player realizes a higher payoff at the expense of the other (Nash, 1950).

Conflicting Lessee and Lessor

Another more complicated example of a two-player game with the value of the remaining reserve as the payoff is a scenario where a lessee is involved in a conflict with a lessor concerning various wells and reserves on the lessee's property. The situation or problem is that a legal action has taken away the lessor's right to recover the reserves from its wells. Therefore, the lessee (L) owns one hundred percent (100%) of the reserves and the lessor/ex-lessor (LEL) owns one hundred percent (100%) of the wells and facilities necessary to recover the reserves and bring them to market. Therefore, the present situation is that the wells have nothing to recover and the reserves have no mechanism to be recovered. So, the parties' mutual cooperation must be considered.

When initially considering the situation, several concepts come to mind. For example, the question that immediately arises is that since L owns the reserves, wouldn't it be more reasonable for it or its agents to operate the wells and facilities rather than LEL? Yet the wells and equipment are still owned by LEL and since neither party would be willing to trust their assets to the other, initially they are not mutually cooperative. Therefore, one of the primary parameters and definitions is that the game is a non-negotiable or non-cooperative one. Or more formally, in game theory jargon, there is no negotiation, but both players have full knowledge of each other's payoff.

There is also consideration as to whether there is a dominant party in this situation. L owns the reserves. The reserves will still have a minimum payoff in the ground. LEL only owns the facilities and wells. If there are no reserves to be recovered, then the value of the wells and facilities are diminished to salvage value only, which is zero, because it would cost more to bring the equipment to market out of the field than their value at market. Therefore, one would initially conclude that L is dominant over LEL, since its minimum payoff is positive and LEL's minimum payoff is negative. This belief will affect the strategies considered by both parties in the game.

Another initial question is "wouldn't it be a more reasonable approach for L to pay LEL for the wells and equipment in the field (which have no value otherwise), because it will be cheaper for L to pay for the existing wells and facilities than to recreate them by building its own?" LEL knows this and would consider that in its strategy for the best payout.

The importance and applicability of this situation to the concept addressed in this paper is that one outcome versus another will affect the value of the reserve to L in all cases and to LEL in some of the cases.

For this example, the first task at hand is to determine the various potential outcomes/strategies, define them and normalize them for the game to be played. As mentioned above, there are two players in conflict, it is initially a non-negotiable game or rather a non-cooperative one, and there appears to be a dominant player and therefore a dominant strategy available for that player. However, as will be seen, all strategies that have positive payoffs for both players are negotiated. Without negotiation, the players could refuse to play and then there is no game. As further explanation, L can decide whether to buy the wells and equipment, rent the wells and equipment, replace the wells and equipment or negotiate a new lease. LEL can decide whether to sell the wells and equipment in place, lease the wells and equipment, remove the wells and equipment or negotiate a new lease. However, unless forced, a player may choose not to proceed with a particular strategy targeted by the other player. This results in zero payoffs for both players. This defined game assumes that the players cannot be forced to accept a particular strategy. Therefore, a dominant strategy could be removed if one player refuses that strategy.

Four outcomes or strategies may be easily identified and defined, one is non-cooperative and the other three are cooperative. The non-negotiated outcome/strategy is that the players remain non-cooperative and LEL plugs and abandons all of its wells and removes all of its equipment. L is left with reserves that are now unrecoverable and LEL foots the bill for the plugging and removal of its wells and equipment. L has to replace the wells and facilities if it wants to recover the reserves, which now have a much diminished value. Without formally assigning payouts in this case, it would appear that this would be the worst outcome for both of the players. Therefore, it may be used as a threat by either party.

The first possible negotiated outcome/strategy is that LEL keeps the wells and facilities and rents them to L. LEL retains an income generating asset and L has a means to recover its reserves. However, there must be substantial mutual trust for this to occur, because either LEL operates the wells recovering L's reserves or L operates LEL's wells and facilities to recover the reserves. Either way, one of the player's has to rely on the other to protect and maintain the other's assets.

The second possible negotiated outcome/strategy is that L buys the wells and facilities from LEL. L has a means of recovering its reserves without having to replace the means of recovering at an anticipated discounted price. LEL rids itself of its plug and abandon liabilities and obtains some value for the wells and equipment in place. Since the maximum value for the wells and equipment is in place and in position to be useful and recover reserves, LEL should, at least, realize the maximum value for the wells and equipment under this outcome. Then L, if the wells and equipment retain mechanical integrity, should realize a substantial discount in its costs required for recovering the reserves.

The third possible negotiated outcome/strategy is that LEL obtains a new lease from the L. Under a new lease, it would be expected that L would receive substantial consideration, since it is the dominating player and its security level is higher than LEL. L's minimum utility is the reserve in place, whereas LEL's minimum utility is zero.

The next step is to assign applicable payouts for the different outcomes and place into a game matrix for analysis. This may be done by assigning appropriate values for individual parameters involved in each outcome. This is somewhat arbitrary based upon the weight of the parameter, rather than actually normalizing. However, it will be seen later that the results support the concept presented herein.

For recoverable reserves (which are defined as those in-the-ground reserves with a means in place for recovery), a value of 4 is given to L since it owns the reserves and a value of 3 is given to LEL since it must acquire the rights to the reserves from L at some cost.

For the wells and equipment in place, a value of 1 is given to L and a value of 2 is given to the ex-lessor/lessor. If the wells and equipment are in place then L realizes value because it has a means of recovering its reserves and LEL realizes value because it has a potential income generating asset.

For renting the equipment, L receives a -1 value since it must now pay LEL for use of its wells and equipment and loses some control in the situation. LEL receives a value of 2 since it has now developed a continuing flow of cash from assets that would be worthless at market and a liability in-place.

For the removal/replacement of the wells and equipment, L receives a value of -3 since it has to replace the removed wells and equipment with new wells and equipment. LEL receives a -2 value since LEL has to remove them.

For the old lease, L receives a value of 2 since it has now rid itself of LEL and retains 100% ownership of the reserves. LEL receives a value of -1 since it now has no right to recover the reserves. This is a good example of how the assigned values may be somewhat arbitrary. It will be shown later in the payoff graph that even if a value of -2 was given to LEL, the analysis would have the same results.

For a newly negotiated lease with LEL, a value of 1 is given to L since it should be able to negotiate favorable terms and it eliminates any costs and substantial risk for recovering its reserves. A value of 1 is given to LEL, since it now has a use for its wells and equipment and retains a partial ownership of the reserves. Only a value of 1 is given in this case, however, because it must be assumed that L would have the upper hand in the derivation of a newly negotiated lease and be able to obtain very attractive terms. Table 2 outlines these assigned values.

Next a game matrix is developed for these four defined outcomes or strategies and each player is assigned a payoff based on the assigned values defined above. For the first outcome/strategy, where the wells and equipment are removed by LEL and replaced by the L, LEL realizes a payoff of -3 and L realizes a payoff of 3 by the summation of the assigned values associated with this strategy. LEL realizes the payoff of -3 by summation of the removal of the wells and equipment (-2) and the termination of the old lease (-1). L realizes a payoff of 3 by summation of the replacement of the wells and equipment (-3), recoverable reserves (4) and the termination of the old lease (2).

For the second outcome/strategy, where LEL retains the wells and equipment and rents them to the lessor, LEL realizes a value of 3 and L realizes a payoff of 5. LEL realizes the payoff of 3 by summation of retaining the wells and equipment in place (2), renting the wells and equipment (2) and the termination of the old lease (-1). L realizes the payoff of 5 by summation of having recoverable reserves (4), having to rent the wells and equipment (-1) and the termination of the old lease (2).

For the third outcome/strategy, where L buys the wells and equipment in place from LEL, LEL realizes a payoff of 1 and L realizes a payoff of 7. LEL realizes the payoff of 1 by summation of obtaining value for the wells and equipment (2) and the termination of the old lease (-1). L realizes the payoff of 7 by summation of having recoverable reserves (4), in place wells and equipment at a discounted price (1) and the termination of the old lease (2).

For the fourth outcome/strategy, where a newly negotiated lease is negotiated, LEL realizes a payoff of 5 and L realizes a payoff of 5. LEL realizes the payoff of 5 by summation of having recoverable reserves (3), having wells and equipment (1) and having a new lease (1). L realizes the payoff of 4 by having recoverable reserves (4) and having a new lease (1). Table 3 outlines the defined matrix for the game.

Figure 2 is the resulting payoff polygon (a triangle in this case with the lease/rental strategy lying within). This result is fairly interesting and would not necessarily come to mind without this analysis. L will, of course, want to retain all rights to the reserves which have been previously proved. However, L would also like to reclassify them as proved producing because this results in a higher fair market value. LEL would like to realize as much value as it can for the wells and equipment and also realize some value represented by the reserves themselves rather than just the wells and equipment used to recover them. Therefore, it makes sense that removing/replacing is the worst strategy for both. Additionally, the strategy of LEL renting the wells and equipment to L results in a higher payoff to LEL at the expense of L and it is actually very inefficient. The strategy of LEL negotiating a new lease with L, results in a lesser payoff to L and a higher payoff to LEL but not necessarily at the expense of the L. This is because L is reducing its risk and having no costs in exchange for a lower payoff.

Lloyd Shapley described one approach to the fair allocation of gains obtained by the cooperation of multiple players, in this case two players, L and LEL. By Shapley's approach, the best negotiated strategy lies somewhere along the line between (1, 7) and (5, 5) and is known as Shapley's Value (Shapley, 1953).

Conclusions

Game theory may be utilized to sort out many uncertainties and questions affecting potential outcomes so that a most likely strategy can be identified and included when estimating the value of the reserves. By doing so, a formal methodology and mechanism is used to determine and analyze the best outcome of the situation and since the outcome affects the value of the reserves, assist in the accuracy of the reserve value estimate.

Probability theory deals with chance, while game theory deals with strategy. Probability theory assists with the classification of the risk associated with the actuality of the reserves amount, while game theory assists with the classification of the strategy associated with the design and implementation of the recovery of these reserves that ultimately defines the value of the reserve.

Acknowledgements

The author would like to thank his wife, Tracey, for her encouragement and suggestions regarding the completion of the paper and Dr. Andrew Wojtanowicz for his encouragement and review of the initial paper concept and abstract.

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Tables

Table 1 – Consent vs Non-Consent Game Matrix	**SI**		
LI		Non-Consent	Consent
	Non-Consent	a, b	c, d
	Consent	e, f	0.1, 0.9

Table 2 – Lessee vs Lessor Values of Parameters	**L**	**LEL**
Reserves	4	3
Wells and Equipment in Place	1	1
Renting Wells and Equipment	-1	2
Removal/Replacement of Wells and Equipment	-3	-2
Old Lease	2	-1
New Lease	1	1

Table 3 – Lessee vs Lessor Game Matrix	**L1 (New)**	**L2 (Rents)**	**L3 (Buys)**	**L4 (Replaces)**
LEL1 (New)	5,5	0,0	0,0	0,0
LEL2 (Sells)	0,0	0,0	1,7	0,0
LEL3 (Leases)	0,0	3,5	0,0	0,0
LEL4 (Removes)	0,0	0,0	0,0	-3,3

Figures

Figure 1 - Probability Theory versus Game Theory

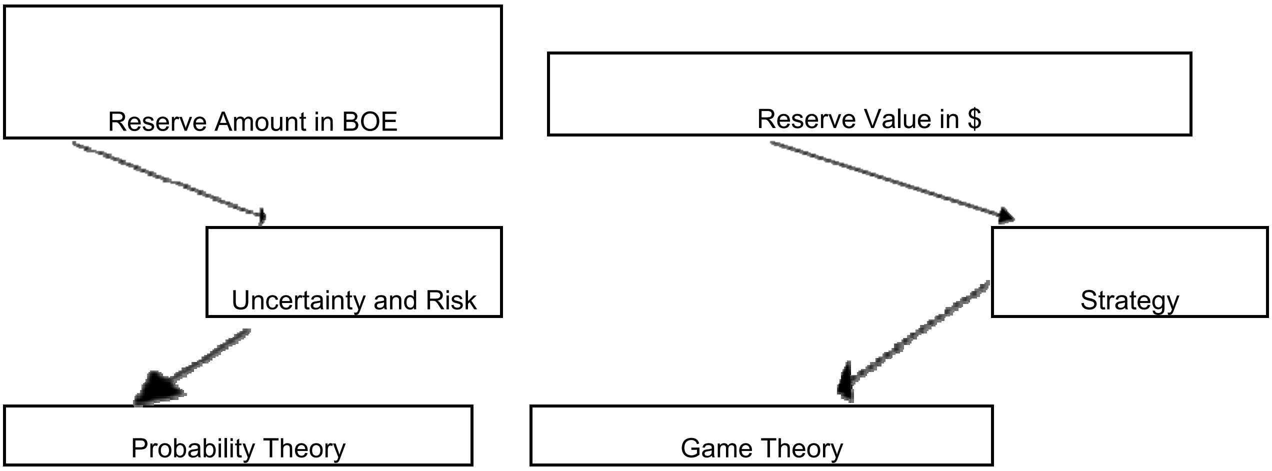

Figure 2 – Lessee vs Lessor Game Matrix Results

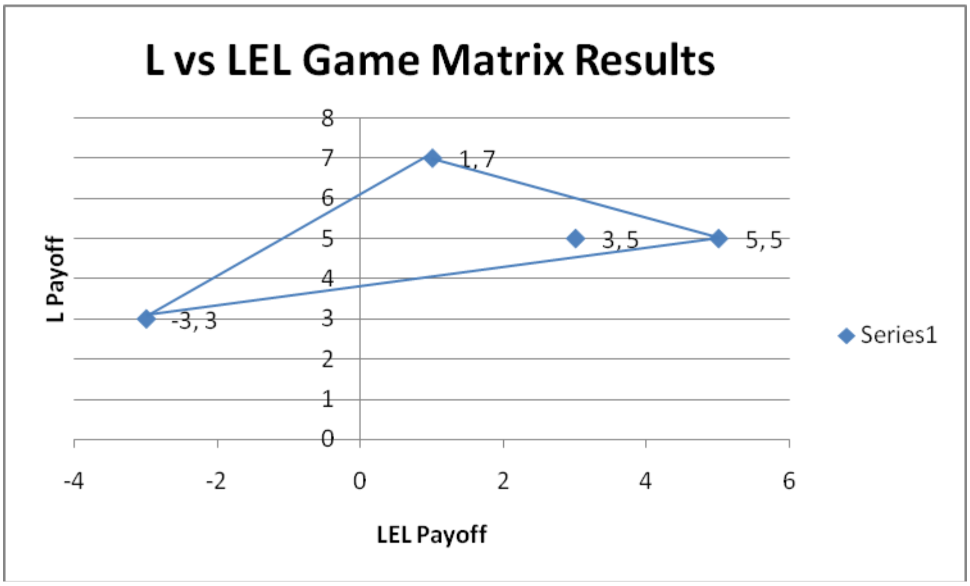

Figure 3 – Small Interest vs Large Interest Game Matrix Results